Research Report Vol. 2, December 2006, Michigan State University

Knowing Mathematics: What We Can Learn from Teachers

In the era of No Child Left Behind (NCLB) a great deal of emphasis has been placed on the importance of a quality teaching force. In fact, states have to certify what percent of their teachers are highly qualified, document how these teachers are distributed across districts, and develop plans for addressing shortfalls (i.e. for reaching 100%) and disparities. The quest for determining what it means for a teacher to be effective, or highly qualified, has been a puzzle for researchers for decades (e.g. Begle, 1972; Begle & Geeslin, 1979; Darling-Hammond, 1999; Darling-Hammond & Youngs, 2002; Good & Grouws, 1987; Zumwalt & Craig, 2005; etc.). Determining what "highly qualified" means involves a mix of empirical information, knowledge of the field, and experience.

Highly Qualified Teachers - Legislation provides a general definition of "highly qualified" and allows states to define some of the specifics. The U.S. Department of Education declares that to be deemed highly qualified, teachers must have: a bachelor's degree, full state certification or licensure, and prove that they know each subject they teach.

At the middle and high school levels, proving that a teacher knows the subject involves demonstrating: a major in the subject they teach, credits equivalent to a major in the subject, passage of a state-developed test, HOUSSE (for current teachers only), an advanced certification from the state, or a graduate degree in the subject they teach.

HOUSSE stands for High, Objective, Uniform State Standard of Evaluation, in relation to which the U.S. Department of Education says, "*NCLB* allows states to develop an additional way for current teachers to demonstrate subject-matter competency and meet highly qualified teacher requirements. Proof may consist of a combination of teaching experience, professional development, and knowledge in the subject garnered over time in the profession." Differences in certification, licensure requirements, and states' HOUSSE standards, lead to varying definitions of a highly qualified teacher across the country (see http://www.ed.gov/nclb/methods/teachers/hqtflexibility.html).

What Mathematics Do Teachers Need to Know?

Recently researchers have been working to determine what mathematics - both general mathematical background, as well as the "specialized" mathematical knowledge (see Ball & Bass, 2000; Ball, Hill, & Bass, 2005; Hill & Ball, 2004; Hill, Rowan, & Ball, 2005) is fundamental to the topics of the school curriculum. Knowing mathematics is necessary certainly for effective mathematics teaching, but researchers, teacher educators, and mathematicians are debating still "what mathematics" is required. And, there is strong agreement that knowing mathematics is not enough to ensure effectiveness in the classroom; teachers need to integrate their mathematical knowledge with knowledge of pedagogy, of students as learners, of curriculum, and of assessment, all within the complex context of schooling.

The mathematical knowledge upon which teachers draw varies depending on whether the teacher teaches at the elementary, middle or high school, because the curricula at these levels vary. Although there do not appear to be any hard and fast standards across teacher preparation institutions about the mathematics needed for the preparation of elementary school teachers, the general belief is that this typically includes knowledge up through college algebra and introductory statistics, including some geometry (often no more than that taken at the high school level), as well as courses in number and arithmetic especially for teachers.

The 2001 *Mathematical Education of Teachers* report of the Conference Board of the Mathematical Sciences (CBMS) makes the following recommendations for mathematics coursework for prospective teachers, noting that the quality of mathematical preparation is more important than the quantity:

1) Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics, including among others:

- number and operations
- algebra and functions
- geometry and measurement
- data analysis, statistics and probability (p.8, pp. 18-23).

2) Prospective middle grades teachers of mathematics should be required to take at least 21 semester-hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers (p.8).

3) Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics in the areas of algebra and number theory; geometry and trigonometry; function and analysis; data analysis, statistics and probability; and discrete mathematics and computer science (p. 8, pp. 40-45).

What Goes On Internationally?

We do not know much about the specific requirements for teachers' mathematical preparation around the world, but we do know several related facts. In many countries secondary school students planning to attend the university are required to take four years of mathematics, often including calculus. Thus students who plan to become primary (elementary) school teachers enter the university with a strong background. This is true in Korea for example. From TIMSS 1995 it has been estimated that U.S. high school students graduate some two or more years behind students in other countries in terms of the level of mathematics they covered (Stigler et al., 1999; Schmidt, Ferrini-Mundy, Houang, & Cogan, 2006). With this strong secondary school preparation as a foundation, future teachers take additional courses at the university level. If we use international data as the benchmark, U.S. primary teachers in general are not as knowledgeable about mathematics as their counterparts in other countries.

At the middle school level, the mathematical knowledge of teachers should be even deeper. In the U.S. there is tremendous variation in the preparation of teachers at this level: some teachers have a major in mathematics, some have a minor, and others have preparation similar to the elementary school teachers. Preliminary data from an MSU-based international study of teacher preparation[1] suggests that middle school teacher preparation in most of the countries studied includes what we in the U.S. would call a *mathematics major*. Teachers take courses in advanced calculus, abstract algebra, analysis, linear algebra and topology.

For the high school teacher the typical requirement around the world, and in the U.S., is a major in mathematics. This has been the case in the U.S. for several decades (see Ferrini-Mundy & Graham, 2004), and is consistent with the NCLB and CBMS recommendations.

What we also have learned about the preparation of mathematics teachers around the world is that preparation in "mathematics-specific pedagogy" (called the "didactics of mathematics" in some countries) is a very significant part of teacher education. In mathematics specific pedagogy courses teachers draw on their knowledge of mathematics and learn about student learning, about curriculum, and about instructional approaches that are related to specific mathematics content areas.

[1] *Developing Subject Matter Knowledge In Mathematics Middle School Teachers: A Cross-National Study Of Teacher Preparation As A Follow-Up of TIMSS (PTEDS).* Supported by NSF Cooperative Agreement REC-0231886.

In this report we examine what the PROM/SE teachers tell us about their knowledge of mathematics and how they acquired it.

What Areas of Mathematics do PROM/SE Teachers Feel Very Well Prepared to Teach?

We asked PROM/SE teachers the following question: "How well prepared academically do you feel you are – that is, you feel you have the necessary disciplinary coursework and understanding – to teach each of the following at the grade level you are currently teaching. . ." for a set of mathematics topics from the K-12 curriculum. The lists of topics for elementary and middle/secondary teachers were different. Here are some highlights.

What do elementary school teachers tell us?

Most elementary school teachers report that they feel very well prepared to teach most of the topics that are part of the curriculum they teach to their students. So why isn't it good enough for teachers to feel very well prepared only on the topics that they teach? Because many of the more advanced topics, for which teachers admit they do not feel well prepared, are necessary as mathematical background for teaching the more elementary topics at their grade level, and for preparing their students for what will come in middle school. For example, less than a quarter of elementary teachers indicate they feel well prepared to teach *proportionality concepts*. Proportionality is fundamental to understanding fractions (an elementary school topic), and to linear functions (an early middle grades topic).

So what topics do elementary school teachers (grades one through five) feel very well prepared to teach? For only two topics do 75% or more of the teachers indicate they feel very well prepared:

- *whole number meaning (including place value),* and
- *operations and properties*.

If we look at only the fourth and fifth grade teachers, 75% also report feeling very well prepared to teach *common fractions*.

When we lower the criterion, and ask for which topics at least half of the PROM/SE teachers indicate they feel well prepared to teach, then eight more topics are included: *common fractions; decimal fractions; relationship between common fractions and decimal fractions; estimation and number sense; measurement units; perimeter, area, and volume; geometry basics;* and *representing and interpreting data* (see Display 1).

Display 1. Percent of Teachers Indicating They Feel Academically "Very Well Prepared" to Teach Each Topic.

Topics (Elementary Teachers)	1-3	4-5	Topics (Middle and High School Teachers)	6-8	9-12
Whole Number Meaning	83	84	Negative, Rational, Real Numbers	65	94
Operations and Properties	77	78	Number Bases	27	62
Common Fractions	63	75	Exponents, Roots, Radicals	54	91
Decimal Fractions	44	64	Complex Numbers	20	73
Relationships between Common and Decimal Fractions	41	62	Number Theory	54	65
Percentages	37	55	Coordinates and Lines	73	95
Properties of Common and Decimal Fractions	31	45	Polygons and Circles	58	82
Number Sets and Concepts	35	38	Three Dimensional Geometry	30	50
Other Number Topics	16	17	Transformations	44	52
Number Theory	19	29	Congruence and Similarity	61	84
Estimation and Number Sense	54	63	Proportionality Concepts	41	82
Measurement Units and Processes	60	59	Proportionality Problems	57	87
Perimeter, Area, and Volume	51	69	Slope	38	92
Estimation and Measurement Errors	33	38	Trigonometry	24	83
Geometry Basics	49	62	Patterns and Relations	53	76
2-D Figures	34	44	Functions	39	85
3-D Geometry	19	22	Expressions and Simple Equations	55	93
Transformations	20	27	Linear Equations and Inequalities	51	94
Congruence and Similarity	39	55	Quadratic and Polynomial Equations and Inequalities	34	90
Proportionality Concepts	13	22	Logarithmic and Trigonometric Equations	10	56
Proportionality Problems	26	31	Systems of Equations and Inequalities	22	75
Slope and Trigonometry	7	5	Representing and Interpreting Data	69	82
Patterns, Relations, and Functions	34	30	Probability and Uncertainty	30	43
Linear Equations and Formulas	15	15	Infinite Processes	11	43
Other Equations and Inequalities	13	12	Change	10	48
Representing and Interpreting Data	45	57	Validation and Justification	6	26
Probability and Uncertainty	27	32	Structuring and Abstracting	N/A	18
Sets and Logic	11	9			

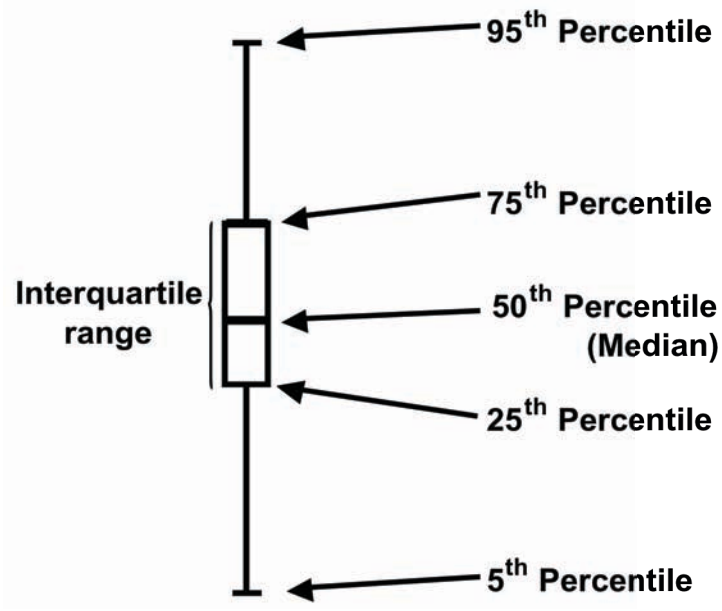
Some key topics are strikingly absent from these lists - topics where it would seem crucial for elementary teachers to have strong understanding. These are: *elementary number theory* (*including integers*); *the rational number system*; *properties of common and decimal fractions*; *primes*; *factors*; *exponents*; and *roots*. These areas form an essential base for understanding the content of elementary topics listed above. We also note that there were almost no geometry topics (except *basics*) for which a majority of elementary grades teachers felt well prepared. And, elementary teachers said they felt unprepared for teaching **all** of the proportionality topics and **all** of the algebra topics.

How does this look for PROM/SE districts at the elementary grades?

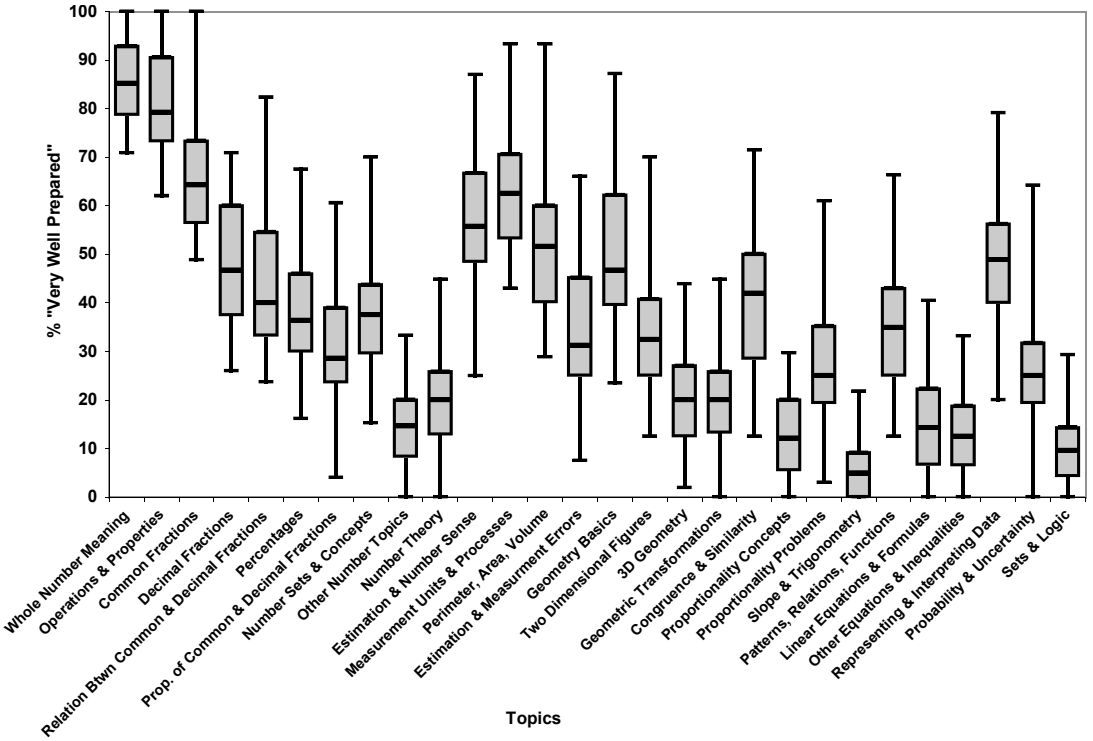
Wouldn't it be ideal if 100% of the PROM/SE teachers felt well qualified to teach at least some of the topics in the elementary curriculum? There were no topics that met this criterion. We assume that within a district a 75% criterion is large enough so that there would be a sufficient knowledge base among the teachers to serve as a resource for those teachers who feel less well prepared. Here we focus on the PROM/SE districts, and use the criterion of 75% to take a look at teachers' feeling of preparedness for each district in PROM/SE. What do we see? These results are given in Display 2.

How to read a Box and Whiskers Plot

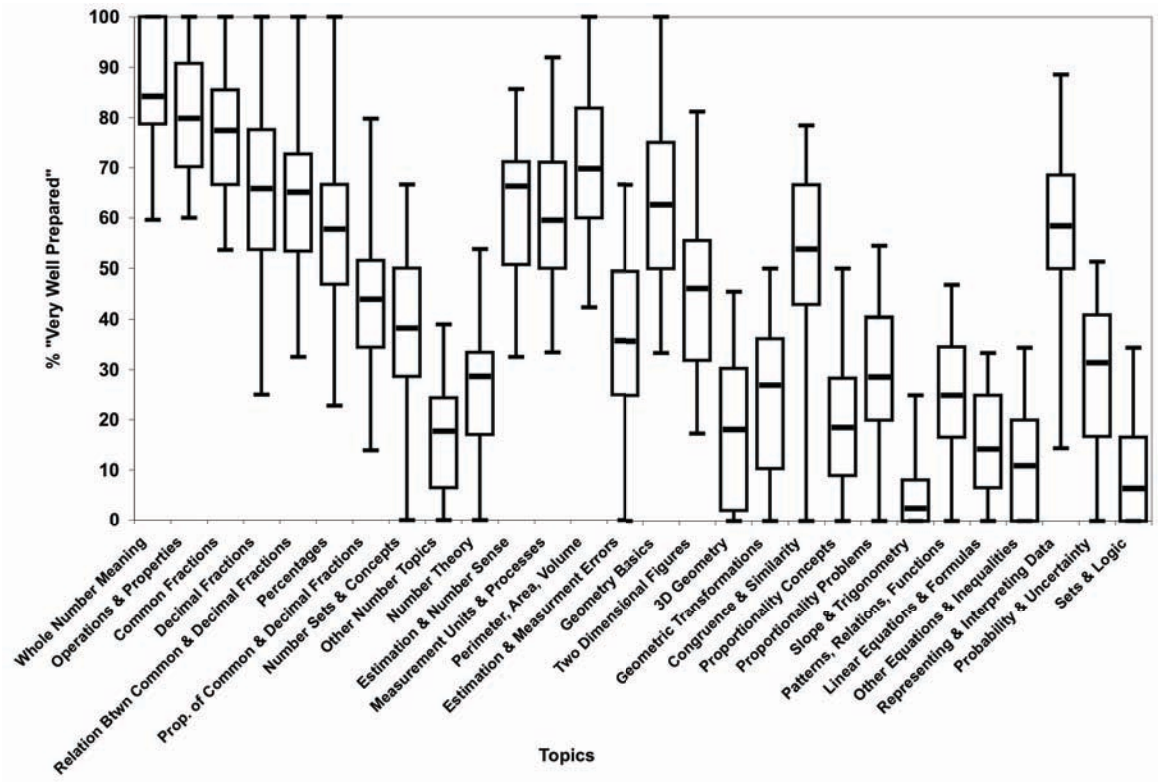
A box and whiskers plot, sometimes called a box plot, provides a visual summary of many important aspects of a distribution. The "box" stretches from the 25th percentile to the 75th percentile, thus containing the middle half of the scores in the distribution. The Median, or 50th percentile, is shown as a line across the "box". The "whiskers" stretch from the 25th and 75th percentiles to the 5th or 95th percentiles, respectively.



Display 2. Distributions across all PROM/SE Districts of the Percent of Lower Elementary (Grades 1-3) Teachers in a District Stating They Feel "Very Well Prepared" to Teach Mathematics Topics.



Display 3. Distributions across all PROM/SE Districts of the Percent of Upper Elementary (Grades 4-5) Teachers in a District Stating They Feel "Very Well Prepared" to Teach Mathematics Topics.



◆ *Observations about teachers of grades one to three* *(see Display 2).*

- For grades one to three there is no topic for which all districts reach the 75% criterion. In other words, even for the topics of *whole number meaning* and *operations* there are some districts (less than 25% for *whole number meaning* and slightly more than 25% for *whole number operations*) in which less than 75% of the elementary school teachers feel very well prepared to teach these topics that are most fundamental to the elementary curriculum.
- Notice that the "typical" value for a district (the median) is below 50% for most of the topics, implying that in at least half of the districts, fewer than half of their elementary teachers claim to feel very well prepared to teach most of the topics listed in Display 1.
- For example, even for *fractions*, which is a topic included in grades two and three in Michigan and Ohio standards documents, less than 25% of the districts meet the criterion of having 75% of their elementary teachers feeling very well prepared to teach *fractions*.
- No district meets the 75% criterion for *decimals* and only a small number meet the criterion for other elementary mathematical areas such as *percentage*, *estimation and number sense*, *geometry basics* and *measurement units and processes*.

◆ *PROM/SE districts vary greatly in teachers' feeling of preparedness.*

- For some districts, all of their teachers report feeling very well prepared to teach *fractions*, while for other districts only about half of their teachers feel this way.
- In the *geometry basics* (e.g., line, angle, etc.), the range goes from a district with only about one-fourth of its teachers feeling very well prepared to another with about 90%.

The consequences of this seem profound both in terms of what a typical student in the PROM/SE districts will encounter and the inequities resulting from the large variation across districts. Table 1 in the appendix gives the results for each PROM/SE district, for teachers of grades one through three.

◆ *A different story for teachers of grades four and five.*

There are eight topics for which there is at least one district in which **all** of the teachers claim to feel very well prepared academically. These are: *whole number meaning*; *operations and properties*; *common fractions*; *decimal fractions*; *relationships between common and decimal fractions*; *percentages*; *perimeter*, *area*, *volume*; and *geometry basics* (see Display 3).

- The results for *whole number meaning* and *operations* are similar to those of

the teachers of grades one through three at the district level. In addition, *common fractions* also has a median value of about 75%.

- A striking feature is the variability across districts. This is particularly large for *decimal fractions*, *percentages* and *geometry basics* - all topics to be introduced in this grade range for Michigan and Ohio.
- Responses about *decimal fractions* have a range across districts, going from a district in which only one-fourth of the teachers feel well prepared to teach the topic to another district in which all teachers feel well prepared. (Basically the same pattern with slight variation holds for the other two topics).

Table 2 in the appendix gives the results for each PROM/SE district, for teachers of grades four and five.

What do middle grades teachers tell us?

In the middle grades, the topics are more advanced, and the teachers' feelings of preparedness are different than those of elementary school teachers.

- In Display 1, there are **no** topics for which at least 75% of the PROM/SE middle grades teachers (grades six through eight) claim to feel very well prepared to teach.
- *Coordinates and lines*, with 73% feeling very well prepared, and *data,* with 69% feeling very well prepared come close.
- There are nine additional topics for which at least 50% of the teachers feel very well prepared: *negative, rational and real numbers*; *exponents, roots and radicals*; *number theory*; *polygons and circles*; *congruence and similarity*; *proportionality problems*; *patterns and relations*; *expressions and simple equations*; and *linear equalities and inequalities*.

The *Michigan Grade Level Content Expectations* and the *Ohio Indicators and Benchmarks* emphasize these topics, in keeping with a strong national movement to include elementary algebra topics, and progressively more sophisticated geometry, in the middle school, culminating in substantial algebra and geometry by eighth grade. These expectations are consistent with international benchmarks in high achieving countries around the world.

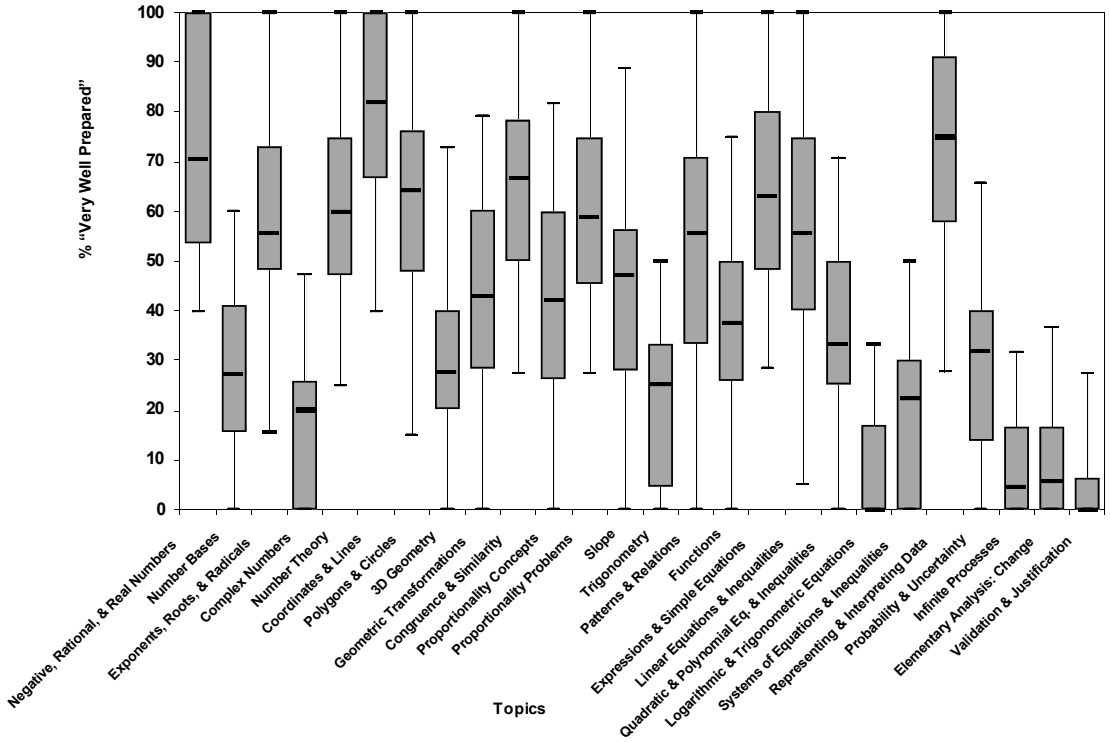
Michigan high school graduation requirements adopted in 2006 specify that students have four credits of high school mathematics to graduate including:

1 credit – Algebra 1
1 credit – Geometry
1 credit – Algebra 2
or 3 credits – Integrated Sequence with same course content

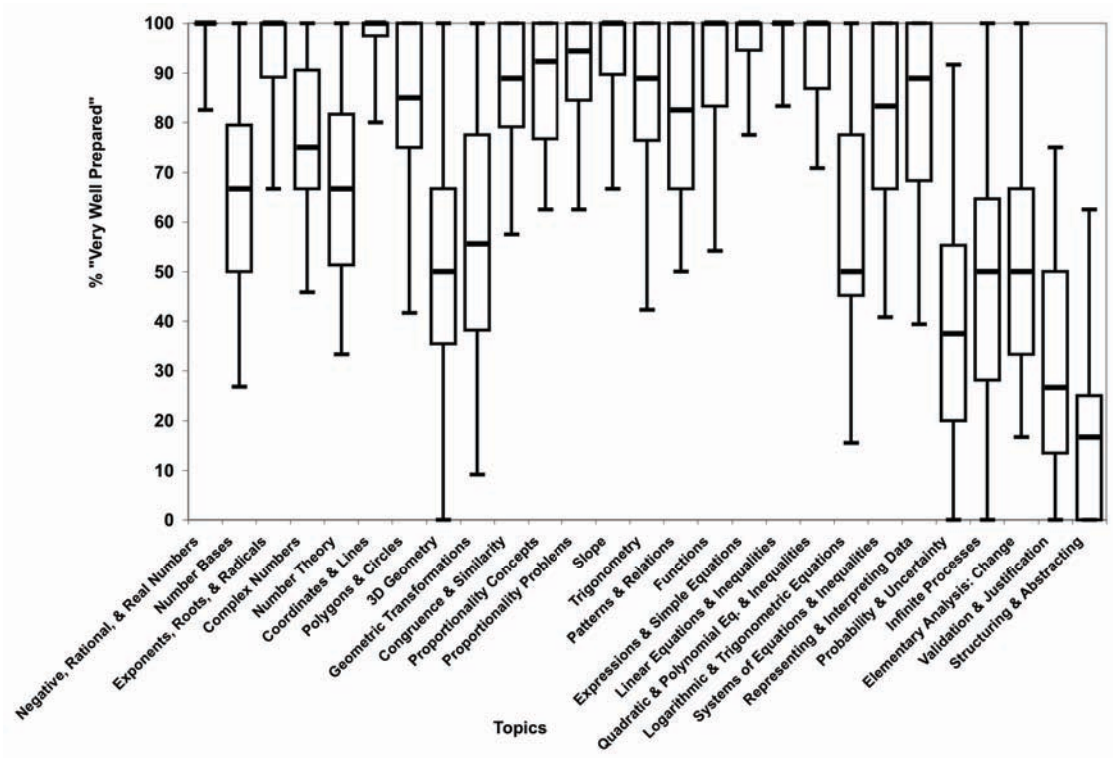
1 additional credit, such as:
Trigonometry, Statistics, Pre-Calculus, Calculus, Applied Math, Accounting, Business Math, retake of Algebra 2

1 credit must be taken the senior year

Display 4. Distributions across all PROM/SE Districts of the Percent of Middle School Teachers in a District Stating They Feel "Very Well Prepared" to Teach Mathematics.



Display 5. Distributions across all PROM/SE Districts of the Percent of High School Teachers in a District Stating They Feel "Very Well Prepared" to Teach Mathematics.



PROM/SE districts face challenges in these curricular areas. Consider these key findings:

- Only about half of the PROM/SE middle grades teachers feel very well prepared to teach *expressions and simple equations* or *linear equations and inequalities*.
- Even fewer PROM/SE middle grades teachers feel very well prepared to teach other important algebraic concepts such as *proportionality* (41%), *slope* (38%), and *functions* (39%).

Because the demographics of the set of PROM/SE districts mirror those of the US as a whole, it is likely that these challenges are even more widespread. The prospects for implementing serious study of algebra in the middle grades are dim unless the issues of teachers' readiness to teach the fundamentals are addressed.

Display 4 shows the district level distributions of middle school teachers indicating they felt "very well prepared" to teach specific mathematics topics. Look at the box-and-whiskers plots for the topics of *coordinates and lines* and *data*. These are the only two topics for which at least one-half of the districts have 75% or more[2] of their teachers at the middle grades that feel their academic background have prepared them very well.

The implication of the results in Display 4 for middle school teachers is like that for the elementary grades teachers, only even more striking.

There is tremendous variation across PROM/SE districts in the degree to which the middle grades teachers feel well prepared academically to teach most of the 26 topics examined. One district has no teachers who feel well prepared to teach *linear equations and inequalities*, while in another district, all of the teachers surveyed report they are very well prepared to teach the topic. Similarly, large variation exists for other fundamental middle school topics such as *exponents, roots, and radicals; number theory; polygons and circles; congruence and similarity; proportionality; slope; patterns and relations;* and *data.* These topics are the core of the more challenging middle school curriculum advocated by Achieve, PROM/SE and the 2004 K-8 Michigan Grade Level Content Expectations. The variation in teachers' feelings of preparedness to teach these topics, across the PROM/SE districts, is very likely to affect

[2] There were no topics that met the 75% criterion for PROM/SE teachers in the aggregate, but this is not inconsistent with these district-level results. In the overall results, both the topics of *coordinates* and *lines* and *data* were close to reaching the 75% level. The result implies that the larger districts likely have lower rates and when data are aggregated over all PROM/SE teachers (where the size of the district is not taken into account) this would make the percent less than the average percent over all districts.

their students' opportunities to learn. The potential for inequity is great.

The district-by-district results for middle school teachers are in appendix Table 3.

What do high school teachers tell us?

The story for high school teachers is quite different from what we learned about elementary and middle school teachers. Almost 60% of the topics in Display 1 meet the "75%" criterion for all PROM/SE teachers combined. The weakest areas in terms of teachers' reported feelings of preparedness include: *number bases* (a topic not taught in U.S. high schools, typically); *three-dimensional geometry; geometric transformations; logarithmic and trigonometric functions; probability;* and *calculus* (*change*). National standards documents from such organizations as the National Council of Teachers of Mathematics (NCTM, 2000), the College Board (2006), and Achieve, Inc. (2004) call for the inclusion of *probability and statistics* at the high school level. With less than half of the mathematics teachers feeling well prepared to teach it, districts face a challenge in this area.

Display 5 gives the district level results for high school teachers. Some highlights:

- There are sixteen topics for which at least 25% of the districts have **all** of their high school teachers indicating they are very well prepared to teach those topics.
- There is a large amount of variation across the districts, especially for geometry topics including *transformations; three-dimensional geometry; polygons and circles; calculus* (*change*)*; probability; number theory;* and *logarithmic and trigonometric functions.*

This means that the problem is not uniform, and thus solutions may need to be district-specific.

Why Do Teachers Feel Ill Prepared?

For teachers of the elementary and middle grades, the answer is simple: in general their preparation is inadequate. The story for some middle school teachers, and for high school teachers, is more complicated.

In this section we summarize what teachers have told us about their preparation in mathematics at the college level and as graduate students.

What kinds of degrees do PROM/SE teachers hold? What about teachers around the world?

We asked PROM/SE teachers (all elementary, middle and high school teachers teaching mathematics) to tell us about their post-high school education – their degrees, year earned, major(s), minor(s), and college or university.

Display 6 indicates the percentage of PROM/SE teachers at each grade level who report having a college major or minor in mathematics. Note that mathematics education majors were coded as having a minor in mathematics; science and engineering majors were included in the "no math specialization" category. Not surprisingly, for teachers of grades one through four, less than 10% report having either a major or minor in mathematics.

Display 6. Percent of PROM/SE Teachers Reporting Mathematics Majors or Minors[3].

Teachers of Grade N	Math Major Reported	Math Minor Reported	No Math Specialization Reported
1	0.6	2.4	97.0
2	0.6	1.8	97.5
3	0.4	1.9	97.7
4	0.5	4.4	95.1
5	1.2	8.6	90.2
6	1.0	8.4	90.6
7	15.8	21.8	62.4
8	18.1	18.5	63.4
9	35.7	26.8	37.5
10	51.2	23.3	25.5
11	36.7	36.7	36.6
12	52.9	27.3	19.8
aggregated across grades			
Grades 1-3	0.5	2.1	97.4
Grades 4-5	0.8	6.3	92.8
Grades 6-8	10.0	14.6	75.4
Grades 9-12	49.7	27.5	22.8

[3] These data are self-reported. Missing data (e.g., question left blank) are excluded from reported percentages. Some teachers indicated "education" as their major, and although mathematics may have been a focus, it was not reported.

International data, even using different definitions, paint a different picture and provide us with a benchmark of sorts. On average, about 30% of the fourth-grade students in the countries assessed in TIMSS 1995[4] had a teacher who held a major or a minor in mathematics or science. Taking this estimate as an indicator of the percentage of teachers with strong mathematics backgrounds, it is considerably higher than for the PROM/SE fourth-grade teachers where the comparable percentage (including teachers reporting science majors/minors) is 14%. In Singapore and Russia, more than 50% of fourth-grade students are taught by teachers holding majors or minors in science or mathematics. This suggests that countries around the world typically have about three times as many elementary teachers who have a specialization in mathematics or in the related field of science. Some additional highlights:

- Three out of four PROM/SE middle school teachers do not report having a specialization in mathematics. This is disturbing because of the higher level of mathematics offered in the middle grades curriculum.

[4] The TIMSS data do not represent a sample of teachers but a sample of students so the indicator is expressed in terms of the percentage of students having a teacher with a major/minor in either mathematics or science. The TIMSS data do not allow an estimation of the number with only a mathematics major or minor.

- Sixth grade teachers seem to have mathematics backgrounds similar to primary teachers - only about 10% report having a major or minor in mathematics. In seventh and eighth grade that percentage increases to about 35 to 40%.

This means that a large percentage of middle school students are being taught the increasingly more complex mathematics called for in the Michigan and Ohio standards by teachers who do not report having a strong background in mathematics. These results could explain why so many middle school PROM/SE teachers do not feel very well prepared to teach many of the middle school topics.

What about high school? We expected to find that almost all of the teachers teaching mathematics would report at least a minor, if not a major, in mathematics. But, here is what we learned:

- Over all PROM/SE high school teachers teaching mathematics courses, about half report having a major in mathematics.
- Almost 25% of high school teachers teaching mathematics report having neither a major nor a minor in mathematics.
- These numbers vary across the four grades taught by the teachers, with over a third of the teachers whose major teaching responsibilities are at the ninth grade not reporting any specializations in mathematics.
- This number reduces to less than 20% for those teachers primarily teaching at the twelfth-grade level.

Recall that on some of the more advanced topics such as *number theory, geometry transformations, logarithmic and trigonometric functions,* and *calculus* there was a sizable proportion - up to half - of the teachers who did not feel very well prepared to teach them. Perhaps these are the same 50% of teachers who do not report having a major in mathematics.

It is somewhat logical to expect that the group of teachers teaching the more advanced courses (usually taken at eleventh and twelfth grades) would be better prepared in mathematics. An alternative view is that it might be even more important for the entry-level high school courses to be taught by the best-prepared teachers. These courses serve as the foundation for further study, may be more difficult to teach effectively, and are crucial in offering equitable opportunities for learning to all students.

Looking more deeply at background: What have PROM/SE teachers studied?

In this section we examine further the question of the backgrounds of the PROM/SE teachers by looking at the mathematics courses they report having taken either as a part of their undergraduate or graduate education. Given that the requirements for a mathematics major or minor can vary across universities, and that not all teachers provided responses to the questions about major and minor, this view of the PROM/SE data provides additional perspective about the type of mathematics preparation PROM/SE teachers have. Display 7 indicates the number of semester hours of mathematics that teachers report having taken at each of the four grade bands. What are some of the key findings?

- For elementary teachers (grades one through five) the typical number of mathematics courses teachers report having taken is about three (assuming one course is equivalent to three semester hours). This is consistent with national recommendations for the preparation of elementary mathematics teachers.
- Middle school teachers report having taken only slightly more than five courses, on average.
- This increases to about eight courses at the high school level.
- The variation across teachers in the middle and high schools is substantial, where the preparation varies from two or three semester-hours to as many as 50 (a variation ranging from one to approximately 15 courses).
- Even among high school PROM/SE mathematics teachers, 25% report having taken less than 18 semester hours of mathematics.

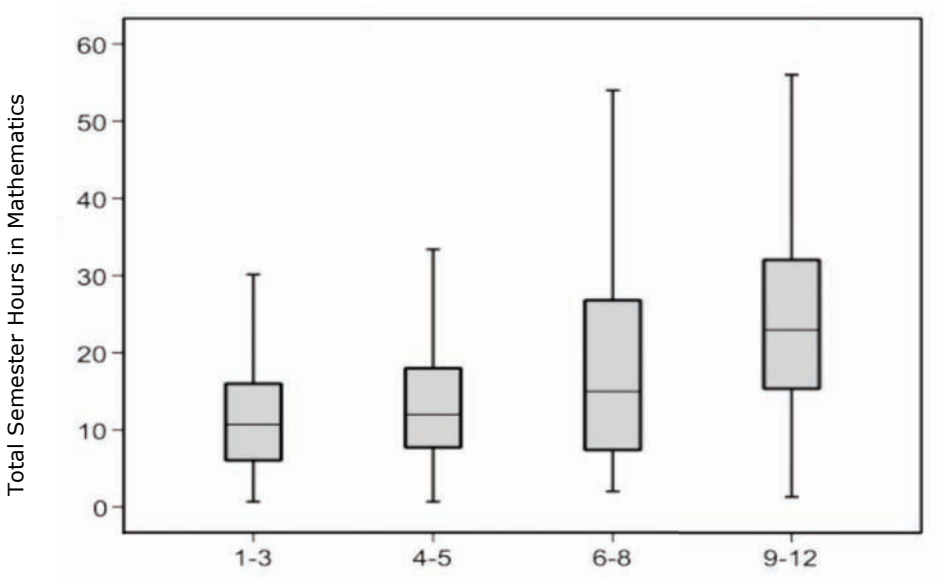
Information about individual courses and the percentage of PROM/SE teachers who have at least one course in given areas is shown in Display 9. What did we find? Some highlights:

- The two courses that teachers most commonly report having taken are *college algebra* and *statistics.* About 40% of the elementary teachers report having taken these courses, and 60-80% of the high school teachers have taken them.
- Very few elementary or middle grades teachers report having taken more advanced courses such as *real analysis*, and only about 50% of the high school teachers report having taken *real analysis*. This might be related to the fact that only about half of them report having a major in mathematics, and/or that majors in mathematics vary greatly in their requirements.

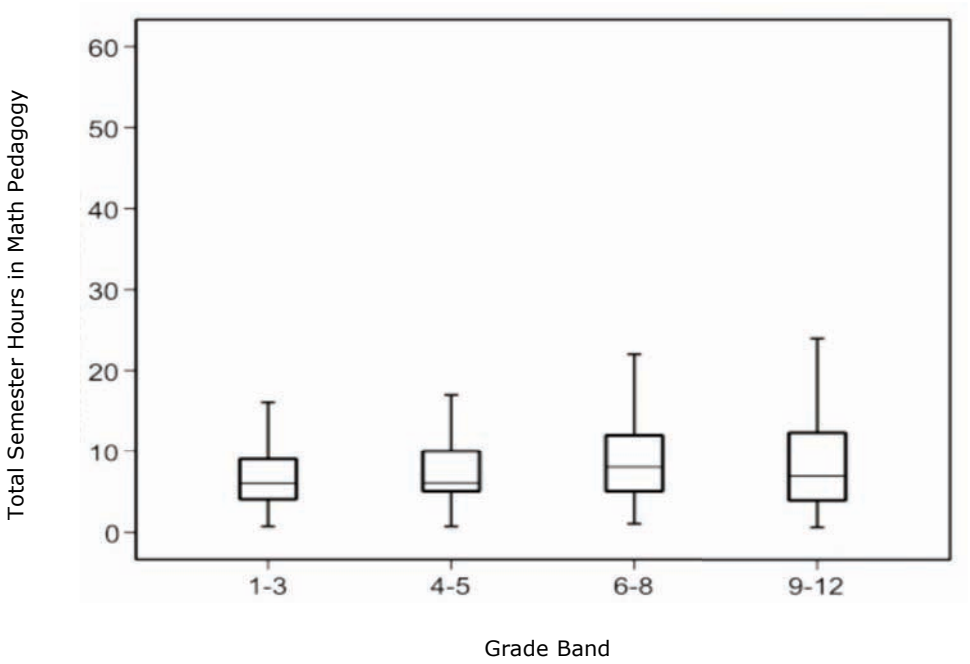
- Reports of *calculus* course-taking vary, from about 15% of the lower elementary teachers to 25% of the upper elementary teachers. These numbers seem surprisingly high, especially since only, at most, 10% of these teachers indicated having completed a mathematics major or minor.
- About half of the middle grades teachers and two-thirds of the high school teachers indicated having taken *calculus*. This would help to explain the earlier result indicating that about half of the high school teachers felt they were not very well prepared to teach *calculus* (*change*).

PROM/SE elementary and middle school teachers have not, generally, specialized in mathematics or taken many mathematics courses. It is not surprising that most of these teachers do not feel very well prepared mathematically to teach the topics they are increasingly being called upon to teach as the new NCLB standards in Michigan and Ohio are more fully implemented.

Display 7. Total Semester Hours in Mathematics Courses, College Algebra and Higher, Reported by PROM/SE Teachers.



Display 8. Total Semester Hours in Mathematics Pedagogy, Reported by PROM/SE Teachers.



Display 9. Number of PROM/SE Teachers Reporting Taking at Least One Semester Credit in Each Course.

	Teachers of									
	Grades 1 - 3		**Grades 4 - 5**		**Grades 6 - 8**		**Grades 9 - 12**		**All PROM/SE Respondents**	
	Count	**Percent**	**Count**	**Percent**	**Count**	**Percent**	**Count**	**Percent**	**Count**	**Percent**
Applied Math	763	44.8	458	44.1	232	38.3	428	49.7	1881	42.2
College Algebra	640	37.6	392	37.8	337	55.6	514	59.7	1883	42.3
Statistics	747	43.9	474	45.7	402	66.3	731	84.9	2354	52.9
Abstract Algebra	178	10.5	151	14.5	171	28.2	699	81.2	1199	26.9
Calculus	278	16.3	243	23.4	320	52.8	557	64.7	1398	31.5
Geometry	346	20.3	257	24.8	295	48.7	662	76.9	1560	35.1
Real Analysis	79	4.6	64	6.2	86	14.2	441	51.2	670	15.1
Topology	50	2.9	36	3.5	38	6.3	162	18.8	286	6.4

What Can We Say about Teachers' Mathematics Knowledge?

Finally we look at a survey of mathematical knowledge that was administered to a sample of the PROM/SE teachers. The primary intention of the survey is to help us in our PROM/SE professional development planning. We asked teachers to solve mathematics and mathematics-for-teaching problems.

The results in Display 10 when combined with those in Display 6 confirm that the elementary and middle school teachers are basically the same. Their responses indicate that they know they are not very well prepared academically to teach the mathematics they are being asked to teach. Display 6 also tells us that over the grades the percent of teachers who do not have majors or minors in mathematics ranges from nearly all of them at grade one to almost 65% at grade eight. Display 10 shows that teachers are able to correctly answer only about half of the items on the survey. This is in comparison to the mathematics majors who at the corresponding grades are able to answer about 70% of the same items correctly. This sizable gap of almost 20% is significant and important; it confirms what teachers tell us when they say they are not very well prepared.

Display 10. Mean Percent Correct on Mathematics-Knowledge-for-Teaching Items in Three Separate Tests for K-6, 6-8, and 9-12 Teachers.

Grade Level Taught	Math Major	Math Minor	No Math Specialization
k-4	67%	61%	49%
5-6	70%	74%	55%
6-8	74%	69%	47%
9-12	74%	70%	64%

At the high school level the problem is more one of variability. As the data indicate most teachers have mathematics degrees, and their performance on the mathematics knowledge survey is reasonable. Still, about one third of the teachers do not have strong academic preparation.

Sample Mathematics Knowledge Item

Which of these students is using a method that could be used to multiply any two whole numbers?[5]

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

[5] Measures copyright 2001, Learning Mathematics for Teaching/Study of Instructional Improvement (SII)/Consortium for Policy Research in Education (CPRE). Not for reproduction or use without written consent of SII. Measures development supported by NSF grants REC-9979873, EHR-0233456 and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

What Are the Long-Term and Systemic Solutions?

Elementary and middle school teachers should be acknowledged for having the insight and integrity to tell us that they do not feel very well prepared to teach mathematics. Their self-reported feelings of preparedness are consistent with their course-taking and performance on a survey of mathematics knowledge.

How can we solve the problem? We should think of this as a systemic problem. Teachers learn much of the mathematics they will ultimately be required to teach while they are students in the K-12 system; college mathematics typically builds on what has come before. So, there is a long-term payoff likely in improving the teaching of mathematics in grades K-12. Part of the problem may well lie with the colleges and universities who educate teachers, decide what courses to require, and plan the curriculum for those courses. And, the requirements of the states that certify the teachers also may need examination. When states raise the bar in terms of K-12 mathematics expectations, teacher education programs may need to consider whether changes are warranted in their curriculum.

One part of the long-term solution is to ensure that teacher preparation programs hold high, explicit mathematical expectations for prospective teachers. Such programs should require adequate study of mathematics, especially for middle school teachers. We must be clear that we do not necessarily mean more courses in advanced mathematics. Rather, research is beginning to indicate that more "mathematics for teaching" – a blend of specialized mathematics content, pedagogical content, and mathematics pedagogy – may be a promising way to improve teachers' mathematics.

The requirements of teacher preparation programs should be made more consistent across institutions. In addition, state certification requirements should also be made more uniform across states to reduce the variability among teachers in the level of their preparation for teaching mathematics. Where middle grades certification programs or teaching endorsements are not available we encourage their development. As a result, and in combination with more uniform standards for students, such policies would help to reduce the inequities in the educational opportunities that permeate the US system.

References

Achieve, Inc. (2004). *The expectations gap: A 50-state review of high school graduation requirements.* Washington, DC: Author.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport CT: Ablex.

Ball, D. L., Hill, H.C, & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, Fall 2005.

Begle, E. G. (1979). *Critical variables in mathematics education.* Washington, DC: Mathematical Association of American and National Council of Teachers of Mathematics.

Begle, E. G., & Geeslin, W. (1972). *Teacher effectiveness in mathematics instruction*. (National Longitudinal Study of Mathematical Abilities Report No. 28). Washington, DC: Mathematical Association of America and National Council of Teachers of Mathematics.

College Board (2006). *College Board Standards for College Success: Mathematics and Statistics.* New York: Author.

Conference Board of the Mathematical Sciences. (2001). *The Mathematical Education of Teachers.* Providence, RI and Washington, DC: American Mathematical Society and Mathematical Association of America.

Darling-Hammond, L. (1999). The case for university-based teacher education. In R. Roth (Ed.), *The role of the university in the preparation of teachers.* (pp. 13 - 30). Philadelphia: Falmer Press.

Darling-Hammond, L., & Youngs, P. (2002). Defining "highly qualified teachers": What does "scientifically-based research" actually tell us? *Educational Researcher*, 31 (9), 13-25.

Ferrini-Mundy, J., & Graham, K. J. (2003). The education of mathematics teachers in the United States after World War II: Goals, programs, and practices. In G. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Good, T. L., & Grouws, D. A. (1987). Increasing teachers' understanding of mathematical ideas through inservice training. *Phi Delta Kappan, 68*(10), 778 - 783.

Hill, H.C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, *35* (5), 330-351.

Hill, H.C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42* (2), 371- 406.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Michigan Department of Education (2006). *Michigan Merit Curriculum High School Graduation Requirements*. Lansing, MI: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Schmidt, W. H., Ferrini-Mundy, J., Houang, R. T., & Cogan, L. S. (2006). *Curriculum insights from PROM/SE.* Unpublished report. East Lansing: Michigan State University, Promoting Rigorous Outcomes in Mathematics and Science Education.

Stigler, J., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). *The TIMSS videotape classroom study: Methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States* (No. NCES 99-074). Washington, DC: U.S. Department of Education. National Center for Education Statistics.

U.S. Department of Education (2004). *Proven Methods, New No Child Left Behind Flexibility: Highly Qualified Teachers, Fact Sheet*. Retrieved November 30, 2006, from http://www.ed.gov/nclb/methods/teachers/hqtflexibility.html

Zumwalt, K., & Craig, E. (2005). Teachers' characteristics: Research on the indicators of quality. In Cochran-Smith, M. & Zeichner (Eds.*) Studying teacher education: The report of the AERA panel on research and teacher education.* Mahwah, NJ: Lawerence Erlbaum Associates.

Appendix Table 1[6]**.** District-by-District Results for Teachers of Grades 1-3.

ID	Whole Number Meaning	Operations & Properties	Common Fractions	Decimal Fractions	Relation Btwn Common & Decimal Fractions	Percentages	Prop. of Common & Decimal Fractions	Number Sets & Concepts	Other Number Topics	Number Theory	Estimation & Number Sense	Measurement Units & Processes	Perimeter, Area, Volume	Estimation & Measurment Errors	Geometry Basics	Two Dimensional Figures	3D Geometry	Geometric Transformations	Congruence & Similarity	Proportionality Concepts	Proportionality Problems	Slope & Trigonometry	Patterns, Relations, Functions	Linear Equations & Formulas	Other Equations & Inequalities	Representing & Interpreting Data	Probability & Uncertainty	Sets & Logic
A	84	73	51	40	40	40	27	44	11	16	51	56	42	27	44	27	11	13	18	11	16	4	22	16	13	33	18	13
AA	91	100	64	64	55	45	45	55	27	45	73	82	82	55	73	64	45	45	45	18	36	9	18	9	9	55	18	18
AB	88	88	50	38	25	25	25	25	0	0	25	63	38	13	38	13	13	0	13	0	13	0	25	0	0	13	0	0
AC	75	75	63	50	25	38	25	25	25	25	63	63	38	25	13	25	38	25	50	25	25	13	50	38	25	63	63	25
AE	100	100	89	67	67	67	56	67	22	33	89	89	89	56	78	67	22	33	67	22	67	22	33	22	22	56	67	11
AF	86	79	64	50	43	43	29	43	36	29	64	64	43	29	43	36	7	21	21	7	21	7	36	29	14	43	36	14
AG	81	94	81	69	69	69	50	44	19	31	75	75	75	63	56	44	38	38	50	31	38	13	44	31	25	56	44	6
AH	72	72	56	28	39	22	17	39	0	11	39	56	50	22	33	39	22	22	50	6	11	0	17	17	17	50	22	6
AI	77	75	54	27	27	33	29	31	8	17	44	46	40	21	40	35	4	10	21	2	8	2	25	10	8	27	13	2
AJ	88	75	50	25	25	25	13	13	0	0	50	63	38	25	50	25	25	25	13	0	25	0	13	0	0	13	13	13
AK	84	84	71	47	46	44	43	40	19	27	65	69	53	37	55	32	25	20	43	17	33	6	43	16	17	52	27	15
AL	71	71	57	29	29	14	0	29	14	14	14	57	29	14	29	14	29	43	29	0	14	0	43	0	0	57	43	14
AM	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
AN	100	91	73	45	36	18	18	36	18	18	73	64	55	27	36	27	18	18	45	9	27	9	27	27	18	45	18	9
AO	97	97	87	74	61	55	58	45	23	26	55	71	52	32	45	26	3	13	42	3	26	0	39	10	3	32	19	3
AP	68	58	48	35	32	26	19	19	6	13	35	39	29	16	26	10	6	10	13	6	19	3	29	6	6	32	29	6
AQ	82	68	53	27	28	20	20	17	2	3	43	65	48	20	35	25	10	8	43	0	5	0	27	7	7	33	22	3
AT	100	100	100	100	100	100	75	75	0	25	75	100	100	50	75	75	25	25	50	25	25	0	50	25	25	75	25	25
AU	93	93	79	64	64	64	64	64	29	29	86	86	86	71	79	86	43	36	79	29	57	21	57	43	29	79	43	36
AV	87	81	70	43	46	48	33	40	22	13	54	67	60	38	56	46	27	16	30	11	24	8	35	11	10	51	29	10
AX	100	100	100	60	40	40	20	40	20	20	80	80	80	60	40	20	20	20	80	20	40	40	80	40	40	80	20	40
AY	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
AZ	67	73	60	53	40	33	27	33	7	13	67	53	60	27	47	20	7	13	40	13	47	0	33	7	7	53	20	0
B	79	66	64	43	34	28	26	31	13	21	43	51	46	16	36	23	16	18	38	10	23	5	25	8	7	41	21	7
BA	83	83	67	33	33	33	33	33	17	0	17	50	33	0	67	17	17	17	50	17	33	0	33	0	0	33	0	0
BB	100	100	63	38	38	13	38	25	13	13	25	38	38	13	38	13	0	0	13	0	0	0	13	0	0	25	0	0
BD	78	78	70	59	49	46	41	38	16	22	62	62	54	27	62	41	24	19	43	22	35	0	35	14	8	46	27	11
BF	83	79	63	38	42	33	25	44	15	25	63	67	50	33	44	33	23	31	60	19	33	4	46	15	10	46	31	10
BG	90	73	57	47	47	30	27	30	20	17	50	63	47	37	40	20	13	13	20	3	23	3	20	17	7	57	20	10
BJ	85	78	70	67	59	52	41	30	7	22	63	56	63	33	63	41	7	26	30	11	22	7	30	11	22	67	22	7
C	84	89	70	51	51	46	30	41	14	24	51	51	57	30	57	32	16	22	51	22	35	8	35	19	19	43	27	8
D	83	78	62	41	38	34	30	33	17	17	51	59	49	33	47	30	16	18	38	12	22	8	33	13	11	40	28	12
E	88	75	63	19	19	13	13	31	13	6	44	50	44	31	31	25	13	13	38	6	13	0	38	19	13	50	19	6
F	70	80	70	60	30	40	30	50	20	20	60	60	60	30	50	30	30	20	60	10	20	0	30	20	20	50	30	0
G	100	100	100	33	33	33	33	33	33	33	67	67	100	33	100	33	0	0	33	0	0	0	0	0	0	67	33	0
H	88	65	65	65	71	59	47	53	24	29	71	71	71	59	65	53	35	24	41	24	41	18	35	41	41	65	47	24
I	89	83	57	49	43	40	34	43	17	23	66	71	57	46	66	46	31	17	46	23	37	14	49	23	20	63	31	17
J	90	83	65	40	39	32	26	38	12	12	55	63	57	26	53	39	13	20	43	8	25	6	35	13	13	52	32	10
K	77	67	49	33	30	25	26	27	17	16	48	47	40	28	50	33	19	22	35	14	22	6	27	13	12	39	19	8
L	82	76	56	44	39	34	31	37	15	18	52	53	52	45	45	39	21	26	45	18	32	11	35	19	13	47	29	15
M	76	65	58	40	38	35	24	29	18	20	51	56	40	35	47	27	25	33	44	16	36	13	42	22	18	45	35	16
N	97	80	80	66	66	60	49	49	29	26	63	63	54	46	51	43	23	20	43	14	26	9	37	14	17	46	31	14
O	100	97	84	66	66	44	34	41	9	16	59	72	56	50	63	41	28	13	56	16	34	6	47	28	19	53	34	13
P	80	80	53	27	13	30	7	17	0	10	60	50	40	20	40	30	17	10	27	0	13	3	30	3	7	30	20	0
Q	89	78	67	56	56	33	33	44	33	44	78	67	67	56	67	56	33	44	33	22	33	22	44	22	22	44	22	11
R	81	78	73	56	48	44	39	38	23	27	56	58	58	41	55	38	31	24	50	18	35	10	46	18	17	49	39	15
S	81	81	71	38	43	33	29	29	14	24	52	48	43	24	43	24	14	10	19	5	19	0	24	0	5	43	19	10
T	75	75	56	44	25	25	25	38	19	25	56	69	63	25	44	38	19	19	13	19	25	13	25	13	19	56	25	13
U	86	90	76	52	48	48	29	43	14	33	76	81	52	43	43	43	14	43	43	24	43	10	57	29	14	52	24	5
V	73	67	45	42	33	36	18	33	9	15	45	64	52	42	52	30	24	21	33	12	24	6	42	15	12	52	30	12
W	74	57	35	22	22	17	9	13	4	9	35	52	39	26	26	13	9	13	17	9	13	4	17	4	4	39	17	4
X	100	80	100	60	60	40	0	40	0	0	40	80	20	0	20	20	20	40	40	0	20	0	40	0	0	40	20	0
Y	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

[6] Data in this table omits districts with 2 or fewer responding teachers. These data were included in Display 2.

Appendix Table 2[7]**.** District-by-District Results for Teachers of Grades 4-5.

ID	Whole Number Meaning	Operations & Properties	Common Fractions	Decimal Fractions	Relation Btwn Common & Decimal Fractions	Percentages	Prop. of Common & Decimal Fractions	Number Sets & Concepts	Other Number Topics	Number Theory	Estimation & Number Sense	Measurement Units & Processes	Perimeter, Area, Volume	Estimation & Measurement Errors	Geometry Basics	Two Dimensional Figures	3D Geometry	Geometric Transformations	Congruence & Similarity	Proportionality Concepts	Proportionality Problems	Slope & Trigonometry	Patterns, Relations, Functions	Linear Equations & Formulas	Other Equations & Inequalities	Representing & Interpreting Data	Probability & Uncertainty	Sets & Logic
A	83	88	75	54	50	54	42	25	4	8	50	63	75	33	58	38	13	8	54	4	13	0	17	8	0	54	17	0
AA	80	75	65	60	50	45	25	50	25	35	60	60	60	25	60	40	40	45	60	35	30	25	35	30	25	55	45	20
AB	100	100	67	33	33	0	0	0	0	0	0	67	67	33	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AC	75	75	50	25	25	25	25	0	0	0	75	50	50	0	25	25	0	0	25	25	0	0	25	0	0	50	25	25
AE	88	88	88	88	88	63	88	50	38	63	88	75	75	88	88	63	38	25	50	25	50	13	38	25	25	50	50	13
AF	67	44	56	44	44	44	0	0	0	33	33	33	44	22	33	22	0	0	11	11	44	0	0	0	0	22	0	0
AG	83	83	75	75	58	75	50	58	42	50	50	58	67	50	75	33	17	25	75	25	42	8	33	25	25	67	33	17
AH	100	100	100	100	100	100	75	50	25	25	75	75	100	25	75	100	25	50	75	75	50	25	25	25	25	75	50	25
AI	85	70	80	65	65	55	45	30	15	30	40	50	70	25	55	35	20	40	50	30	30	5	25	15	10	60	10	5
AJ	100	67	83	50	50	83	33	17	17	0	67	33	67	17	100	50	67	17	50	0	0	0	0	0	0	50	0	0
AK	85	77	77	67	59	54	51	36	18	28	69	51	59	31	59	38	15	5	38	21	21	3	21	5	8	54	15	8
AL	86	71	86	86	86	57	43	29	0	29	100	86	86	57	86	71	43	43	86	29	29	0	43	14	14	86	43	0
AM	83	100	100	100	83	67	83	50	33	33	83	83	83	33	100	83	50	50	67	33	50	17	17	17	17	67	50	50
AN	91	64	82	73	73	73	45	18	18	18	55	55	82	18	55	45	18	45	55	18	27	0	18	9	18	73	36	0
AO	100	100	100	94	83	83	78	78	22	39	61	61	89	50	78	56	28	28	67	22	44	6	28	28	39	67	50	33
AP	92	92	83	67	58	58	50	42	33	33	67	50	83	67	67	58	33	50	50	17	58	8	25	17	8	67	33	8
AQ	77	68	68	59	73	41	23	18	5	27	45	59	68	18	59	32	23	36	50	18	23	5	18	9	5	73	32	0
AS	100	100	100	100	67	67	67	33	33	67	67	100	100	33	100	67	33	67	67	33	33	0	33	33	33	100	33	33
AT	100	91	82	91	82	73	55	55	36	55	82	91	91	73	73	55	36	36	73	55	55	18	55	27	36	73	55	36
AU	80	67	67	53	53	67	40	60	20	27	53	60	60	33	73	47	13	27	60	20	53	13	33	27	20	67	27	7
AV	78	78	78	61	65	61	43	57	17	48	57	65	70	48	74	61	35	35	74	26	35	9	48	30	26	74	35	17
AX	83	67	67	67	67	67	67	67	50	50	50	67	50	50	50	50	50	50	67	50	50	33	17	33	17	83	33	17
AY	67	67	67	67	67	67	67	33	0	33	67	33	33	0	33	0	0	0	33	0	0	0	0	0	0	33	0	0
AZ	78	83	72	78	56	61	50	44	22	28	72	50	72	33	72	56	22	22	44	28	33	6	33	39	22	67	17	0
B	90	82	73	60	62	53	42	32	10	25	48	42	57	32	48	30	13	23	43	15	25	3	18	12	7	52	25	5
BA	83	67	83	83	83	50	67	50	33	33	83	67	83	67	83	50	33	33	50	50	50	33	33	50	50	83	33	33
BB	78	78	78	67	67	67	11	33	22	11	56	67	67	44	67	44	0	33	56	0	0	0	33	0	11	56	33	0
BD	71	61	64	54	54	46	32	29	7	32	57	39	61	29	54	32	25	36	54	25	25	4	18	4	7	50	29	4
BF	100	88	88	69	63	56	38	25	6	19	69	94	75	19	63	44	19	25	81	6	38	0	25	6	6	94	38	6
BG	100	93	79	71	79	79	50	57	29	29	79	71	86	50	71	64	14	7	57	29	29	14	36	29	14	64	29	7
BJ	85	77	85	77	77	69	62	54	15	31	85	85	77	46	77	62	23	38	77	31	38	8	46	23	23	69	54	38
C	88	71	63	54	42	42	29	38	13	17	50	50	50	13	54	13	8	8	42	8	13	4	25	17	13	38	13	4
D	81	78	75	64	61	56	48	35	21	34	66	61	67	41	63	48	23	34	53	28	29	5	37	15	13	60	47	9
E	46	62	46	23	31	23	15	15	8	8	31	38	38	15	38	23	15	8	23	8	15	0	15	0	0	31	15	8
F	87	80	80	53	67	60	47	33	13	53	80	53	73	53	67	53	33	40	67	13	40	7	40	13	20	60	60	27
G	100	100	71	71	71	86	43	43	0	29	71	57	57	43	57	43	14	29	43	14	29	0	43	14	0	57	29	14
H	100	100	100	100	100	100	40	40	0	0	40	20	80	20	40	20	0	20	60	20	0	0	20	20	20	40	20	20
I	67	78	67	56	56	22	33	11	11	11	67	78	89	56	78	67	11	0	67	0	22	0	11	0	0	44	11	0
J	97	85	88	61	70	52	48	36	15	27	64	88	82	45	64	36	18	39	64	18	36	0	39	9	3	64	42	6
K	81	67	70	55	56	48	41	31	19	31	64	56	66	42	61	48	22	25	58	17	22	5	23	8	11	48	17	5
L	83	83	76	69	71	62	33	33	21	29	57	55	64	38	64	43	19	36	60	19	36	2	21	10	10	55	31	12
M	89	83	89	61	61	56	39	39	17	17	67	67	83	39	78	50	28	28	67	17	28	11	22	11	11	72	39	11
N	100	91	91	82	82	82	73	64	18	45	82	91	100	64	91	73	18	27	82	18	55	9	36	18	18	82	36	9
O	100	100	100	100	100	100	100	67	33	33	67	100	100	33	100	100	33	33	67	33	67	0	33	33	0	100	33	0
P	59	59	59	53	41	41	47	47	12	24	53	53	59	24	35	24	0	18	53	24	24	0	24	12	12	47	18	6
Q	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
R	84	80	74	69	66	54	52	43	19	29	70	59	74	45	66	49	31	28	60	30	40	6	44	17	14	55	41	10
S	80	80	80	20	40	20	40	40	0	0	60	80	80	40	80	80	0	20	40	0	20	0	0	20	20	60	20	0
T	71	71	71	43	57	43	43	14	0	0	43	43	71	43	43	29	0	29	43	14	14	0	14	14	0	57	43	0
U	100	90	80	60	70	60	20	40	20	10	70	70	70	40	50	50	0	20	10	0	20	0	10	10	0	50	10	0
V	83	78	61	50	50	50	44	44	22	28	67	50	56	28	33	28	17	17	33	17	22	0	11	22	6	50	22	0
W	60	80	60	40	40	40	40	0	0	0	20	40	60	0	40	20	0	0	0	0	0	0	0	0	0	0	0	0
X	100	100	100	100	67	67	67	67	0	33	67	67	67	67	33	33	0	0	0	33	33	0	0	33	0	67	0	0
Y	50	25	50	25	25	25	25	25	25	25	50	50	25	50	50	25	0	0	50	0	0	0	50	0	0	50	25	25

[7] Data in this table omits districts with 2 or fewer responding teachers. These data were included in Display 3.

Appendix Table 3[8]**.** District-by-District Results for Middle School Teachers of Grades 6-8.

ID	Negative, Rational, & Real Numbers	Number Bases	Exponents, Roots, & Radicals	Complex Numbers	Number Theory	Coordinates & Lines	Polygons & Circles	3D Geometry	Geometric Transformations	Congruence & Similarity	Proportionality Concepts	Proportionality Problems	Slope	Trigonometry	Patterns & Relations	Functions	Expressions & Simple Equations	Linear Equations & Inequalities	Quadratic & Polynomial Eq. & Inequalities	Logarithmic & Trigonometric Equations	Systems of Equations & Inequalities	Representing & Interpreting Data	Probability & Uncertainty	Infinite Processes	Elementary Analysis: Change	Validation & Justification
A	53	33	33	27	47	53	40	20	33	40	33	47	27	27	40	27	47	40	27	27	27	60	27	13	20	7
AA	60	40	65	25	55	80	70	30	55	70	45	50	50	30	50	55	70	60	35	25	30	65	30	15	15	10
AB	67	33	67	0	67	67	33	0	33	67	67	67	33	0	33	33	67	67	33	0	0	67	33	0	0	0
AC	100	0	33	0	33	67	33	0	0	33	0	33	0	0	33	0	67	0	0	0	0	67	33	0	0	0
AE	50	25	50	50	50	100	75	25	25	25	25	50	50	50	25	25	50	50	25	0	25	50	0	0	0	0
AF	43	14	43	14	71	71	86	29	43	43	29	43	29	14	57	14	71	29	14	14	14	43	43	14	14	14
AG	20	0	20	0	0	40	20	0	0	0	0	0	0	0	20	0	0	0	0	0	0	20	0	0	0	0
AH	75	50	50	25	75	75	75	50	25	100	75	75	25	0	75	75	50	75	25	0	50	75	50	0	0	0
AI	71	18	59	29	65	82	53	24	59	71	41	59	47	41	47	29	41	53	29	6	29	82	35	18	24	6
AJ	50	0	50	0	50	50	0	0	50	50	0	50	0	50	0	0	50	0	0	0	0	0	0	0	0	0
AK	68	42	63	42	47	89	79	26	37	79	42	89	58	26	79	58	63	74	53	11	32	95	32	16	5	5
AL	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
AM	80	60	60	40	60	100	90	30	30	70	80	90	70	50	70	50	90	90	60	20	30	80	40	20	30	20
AN	100	50	100	17	83	100	83	83	83	100	67	67	50	17	100	33	83	83	33	0	17	100	33	17	17	0
AO	100	50	100	25	75	100	75	50	50	75	25	75	75	25	75	50	100	50	75	0	25	100	25	25	25	25
AP	100	75	100	0	100	100	100	25	75	100	75	100	50	25	100	50	100	75	25	0	0	100	0	25	25	0
AQ	89	28	83	22	56	83	78	44	61	72	61	72	56	44	72	67	67	78	56	6	39	89	39	6	17	6
AS	100	50	100	50	75	100	50	50	75	75	75	75	75	50	75	75	100	100	100	50	50	75	50	25	50	0
AT	100	33	100	33	67	100	100	33	67	100	100	100	100	33	67	100	100	100	67	33	67	100	33	33	33	33
AU	36	27	27	9	45	55	45	27	27	55	18	45	9	9	36	9	27	36	36	0	18	36	18	0	0	0
AV	64	21	71	14	57	86	64	36	50	64	57	79	64	43	57	57	71	64	57	21	50	71	43	0	7	7
AX	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
AY	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
AZ	40	20	20	20	60	60	60	20	20	60	20	40	40	20	20	40	40	40	40	20	20	40	40	20	20	0
B	73	32	59	18	59	82	50	18	36	55	27	45	36	23	64	36	55	50	32	5	18	77	36	5	14	0
BA	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
BB	50	33	50	33	67	67	67	33	67	50	67	67	50	33	50	33	67	33	33	33	33	83	33	33	33	33
BD	57	29	46	25	39	79	64	14	61	68	36	61	39	36	57	46	50	50	43	11	25	64	21	18	7	11
BF	67	0	11	0	22	67	67	33	56	44	44	44	33	22	33	22	33	56	22	0	0	56	11	0	0	0
BG	67	22	56	0	56	67	22	11	22	44	22	33	44	11	33	22	33	33	44	11	22	22	22	0	0	11
BJ	100	50	67	50	100	100	100	83	83	83	83	83	67	33	83	67	83	83	67	0	33	83	17	17	17	0
C	72	17	56	6	28	78	67	28	28	67	56	83	56	33	61	44	67	67	33	6	28	83	17	17	6	6
D	56	25	42	14	46	63	51	27	42	55	34	46	23	12	47	38	42	38	24	8	11	60	32	11	7	6
E	78	11	67	11	44	89	56	22	33	78	56	67	56	33	56	33	56	56	33	11	22	67	11	0	0	0
F	40	10	30	20	30	40	30	10	10	30	10	30	20	0	20	30	40	40	20	0	20	50	10	10	0	0
G	100	60	100	20	100	100	100	20	60	100	80	100	40	0	60	40	100	60	20	0	20	100	60	0	0	0
H	100	33	67	0	33	100	33	33	33	67	33	33	0	0	33	33	33	33	0	0	0	33	33	0	0	0
I	100	43	86	14	86	86	57	29	29	71	57	71	57	29	57	43	71	71	43	29	43	100	71	14	14	14
J	53	13	53	27	67	67	67	40	60	67	53	67	47	47	67	40	60	53	40	27	33	93	40	13	20	7
K	59	23	52	25	55	66	54	38	36	46	39	55	32	30	48	34	55	45	29	11	25	66	36	11	7	5
L	63	38	75	13	75	75	75	63	63	75	50	50	38	13	75	38	75	50	25	0	13	75	38	13	13	0
M	100	80	80	40	80	100	80	40	60	80	60	100	40	20	80	80	80	80	60	0	20	100	60	0	0	0
N	80	40	70	20	70	100	70	60	70	100	60	70	60	50	70	40	80	70	50	10	30	80	60	30	10	30
P	50	20	50	20	40	30	10	20	30	40	30	30	10	10	40	20	30	10	10	0	0	40	0	0	0	0
Q	75	0	0	0	0	75	25	0	50	50	0	25	0	0	50	25	25	25	25	0	0	75	0	0	0	0
S	89	22	78	44	89	89	78	0	67	78	33	33	78	0	33	67	33	78	56	22	33	89	0	0	0	0
T	75	0	25	25	50	50	50	25	25	50	0	50	0	25	75	25	50	50	50	0	0	75	25	0	0	0
U	60	40	60	20	60	40	40	40	40	60	40	60	60	20	60	40	60	60	20	20	20	80	20	0	40	0
V	73	27	55	27	82	82	73	55	27	55	27	55	27	27	27	27	64	64	36	0	27	82	27	0	0	0
W	100	25	100	25	100	100	50	25	75	100	50	100	50	25	100	50	100	75	50	0	25	100	75	0	0	0
X	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Y	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

[8] Data in this table omits districts with 2 or fewer responding teachers. These data were included in Display 4.

Promoting Rigorous Outcomes in Mathematics and Science Education (PROM/SE) is a comprehensive research and development effort to improve mathematics and science teaching and learning in grades K-12, based on assessment of students and teachers, improvement of standards and frameworks, and capacity-building with teachers and administrators.

PROM/SE Partners:
Calhoun ISD, MI
High AIMS Consortium, OH
Ingham ISD, MI
Michigan State University
SMART Consortium, OH
St. Clair County RESA, MI

Co-Principal Investigators:
Joan Ferrini-Mundy
William Schmidt
Peter Bates
Terry Joyner
George Leroi
Chris Wigent

The PROM/SE RESEARCH REPORT is published by PROM/SE, Michigan State University, 236 Erickson Hall, East Lansing, MI 48824.

Send inquiries to:
Susan Pettit Riley,
Dir. of Communications
(517) 353-4884,
promse@msu.edu
www.promse.msu.edu

The PROM/SE RESEARCH REPORT is based on work supported by the National Science Foundation under agreement EHR-0314866. Any opinions, findings and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the NSF.

What Can School District Leaders Do Now?

Below are some steps that school district leaders can take right away to support teachers who are recognizing that their mathematical content knowledge needs bolstering.

- Acknowledge that focus on instructional materials and mathematics pedagogy is important but alone is unlikely to lead to significant improvement in students' mathematics performance.

- Recognize that teachers need professional development opportunities that are focused on specific topics in the mathematics school curriculum to offer them a "profound, fundamental understanding" (Ma, 1999) of those topics. Use PROM/SE data to help identify areas of need.

- Understand that episodic, "one shot" professional development offerings, and generic professional development, are not likely to help teachers develop a coherent picture of how important mathematical concepts unfold across the grades. Curriculum for K-12 students should be mathematically coherent; likewise, professional development for teachers should also be mathematically coherent.

- Sometimes it will be difficult to hire teachers with adequate content background. Create and implement induction programs for beginning teachers that emphasize the teaching of specific mathematics content.

- Undertake aggressive recruitment and retention strategies, particularly for middle grades teachers who have a mathematics background, and ideally a major in mathematics. Seek out high school teachers whose primary undergraduate area of study was mathematics.

- Do not assign "out of field" teachers to mathematics courses. Assign some of the most mathematically sophisticated teachers to foundational high school courses such as first-year algebra.

- Give teachers opportunities to "retool" with short courses, on-line experiences, and workshops in areas of the curriculum that are emerging with more emphasis, such as *probability and statistics, functions, calculus*, and *transformational geometry*.

Teaching mathematics effectively is an enormously challenging task. PROM/SE teachers have the commitment and interest to continue to build their knowledge and skills to accomplish this challenge. The wise and foresightful support of district administrators is essential to this improvement.